Definitions

- **Network** – Any structure containing interconnected elements.
- **Circuit** – Usually physical structure constructed from electrical components.

(A) **Linear Network**: response proportional to excitation. Superposition applies:

$$If \quad e_1(t) \to w_1(t) \quad and \quad e_2(t) \to w_2(t)$$

Then

$$k_1 \cdot e_1(t) + k_2 \cdot e_2(t) \to k_1 \cdot w_1(t) + k_2 \cdot w_2(t)$$

(B) **Time-Invariant Network**: $e(t) \to w(t)$ relation the same if $t \to t + t_1$. Time varying otherwise.

(C) **Passive Network**: EM energy delivered always non-negative. Specifically:

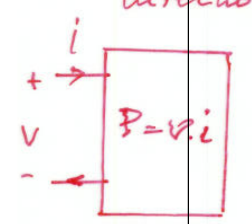
$$E(t) = \int_{-\infty}^{t} v(x)i(x)dx \geq 0$$

or

$$E(t) = \int_{t_0}^{t} v(x)i(x)dx + E(t_0) \geq 0$$

This must be true for any voltage and its resulting current for all t

Associated directions:



In general

$$\sum_j \int v_j i_j dx \geq 0$$

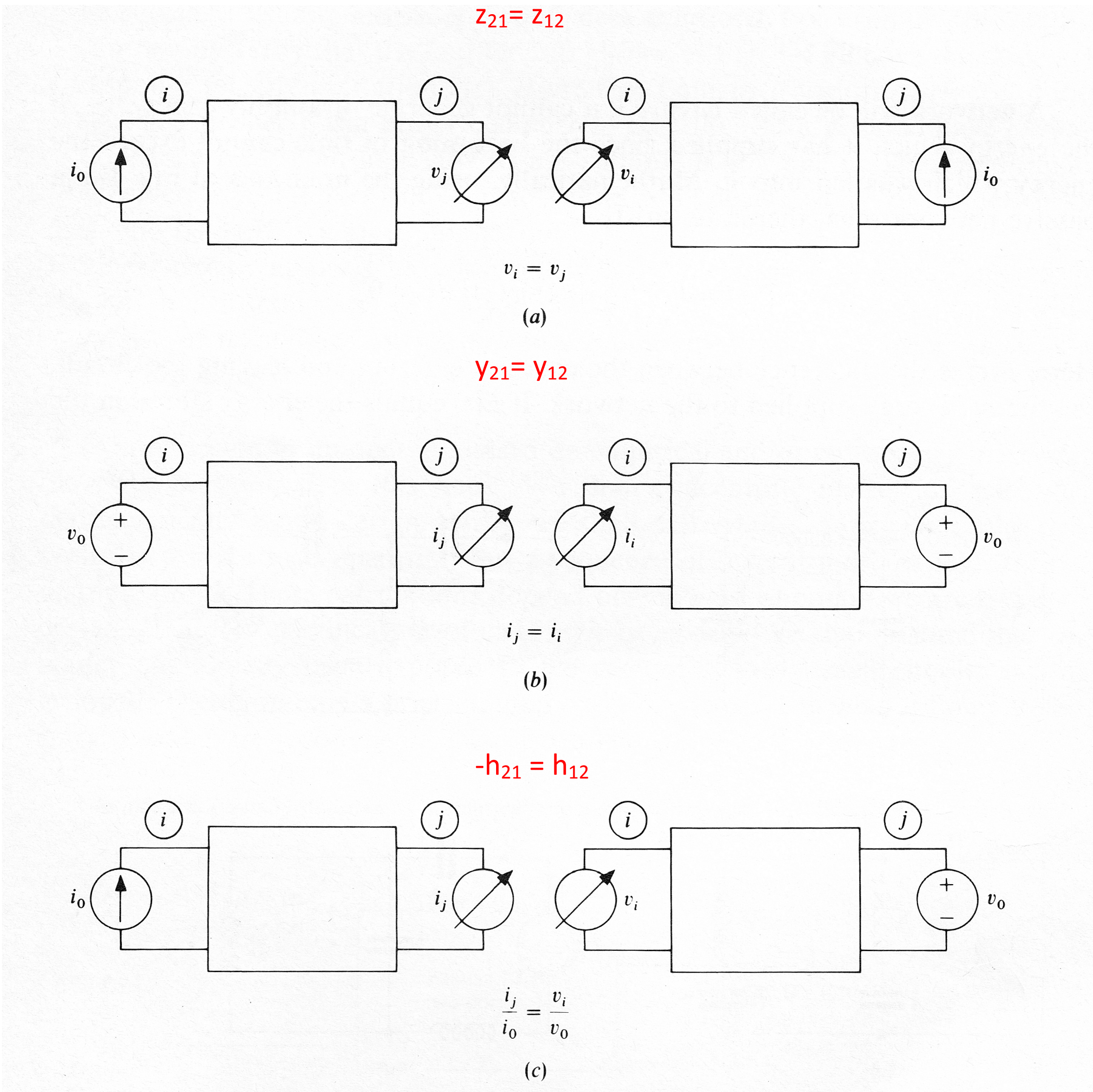
Otherwise, active.

(D) **Lossless Circuit**: input energy is always equal to the energy stored in the network. Otherwise, lossy.

(E) **Distributed Network**: must use Maxwell's equation to analyze. Examples: transmission lines, high speed VLSI circuits.

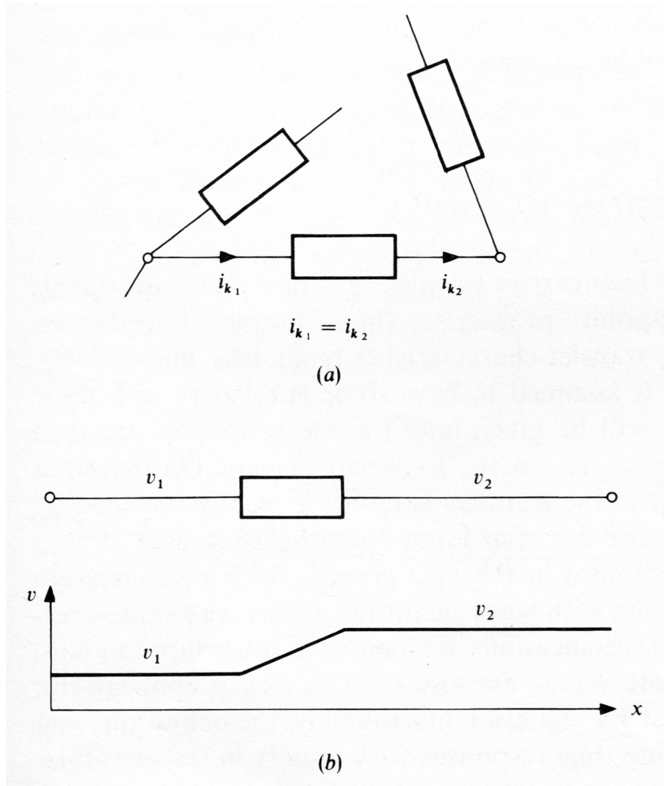
(F) **Memoryless or Resistivity Circuit**: no energy storing elements. Response depends only on instantaneous excitation. Otherwise, dynamic or memoried circuit.

(G) **Reciprocity**: response remains the same if excitation and response locations are interchanged. Specifically:



Otherwise, non-reciprocal.

(H) **Lumped Network**: physical dimensions can be considered zero. In reality, much smaller than the wavelength of the signal.



(I) **Continuous-Time Circuit**: the signals can take on any value at any time.

(J) **Sampled-Data Circuit**: the signals have a known value only at some discrete time instances. Digital, analog circuits.

> An ideal RLC circuit is linear, time-invariant, passive, lossy, reciprocal, lumped, dynamic continuous-time network.

(A) **Ideal R, L, C**:

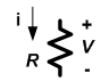
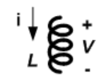
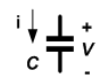
		Voltage-Current Relationship		
Element	**Parameter**	**Direct**	**Inverse**	**Symbol**
Resistor	**Resistance *R*** **Conductance *G***	$v = Ri$	$i = \frac{1}{R}v = Gv$	
Inductor	**Inductance *L*** **Inverse Inductance *T***	$v = L\frac{di}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(x)dx + i(0)$	
Capacitor	**Capacitance *C*** **Elastance *D***	$i = C\frac{dv}{dt}$	$v(t) = \frac{1}{C}\int_0^t i(x)dx + v(0)$	

Table 1

Each passive.

Assuming standard references, the energy delivered to each of the elements starting at a time when the current and voltage were zero will be:

$$E_R(t) = \int_{-\infty}^{t} Ri^2(x)dx \geq 0 \qquad (67)$$

$$E_L(t) = \int_{-\infty}^{t} L\frac{di(x)}{dx}i(x)dx = \int_{0}^{i(t)} Li'di' = \frac{1}{2}Li^2(t) \geq 0 \qquad (68)$$

$$E_C(t) = \int_{-\infty}^{t} C\frac{dv(x)}{dx}v(x)dx = \int_{0}^{v(t)} Cv'dv' = \frac{1}{2}Cv^2(t) \geq 0 \qquad (69)$$

(B) **Ideal Transformer**:

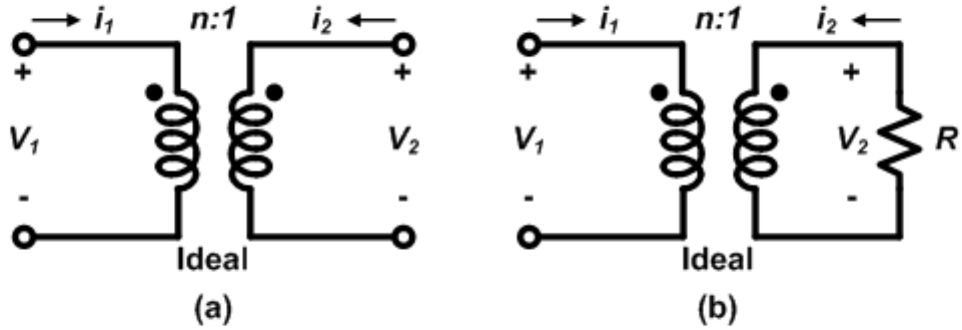


Fig. 6 An ideal transformer

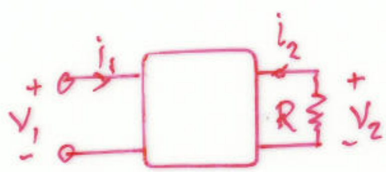
Defined in terms of the following v-i relationships:

Memoryless

$$v_1 = nv_2 \tag{70a}$$
$$i_2 = -ni_1 \tag{70b}$$

or

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & n \\ -n & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} \tag{70c}$$



$$v_1 = nv_2 = -nRi_2 = (n^2R)i_1 \tag{71}$$

At the input terminals, then, the equivalent resistance is n^2R. Observe that the total energy delivered to the ideal transformer from connections made at its terminals will be

$$E(t) = \int_{-\infty}^{t} (v_1(x)i_1(x) + v_2(x)i_2(x))dx = 0$$
$$P = 0 \tag{72}$$

Lossless, memoryless!

The right-hand side results when the v-i relations of the ideal transformer are inserted in the middle. Thus, the device is passive; it transmits, but neither stores nor dissipates energy.

Memoryless!

(C) **Physical Transformer**:

L_1: primary self-inductance

M: mutual inductance

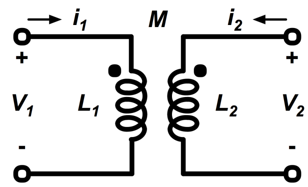


Fig. 7 A transformer

The diagram is almost the same except that the diagram of the ideal transformer shows the turns ratio directly on it. The transformer is characterized by the following v-i relationships for the reference shown in Fig. 7:

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \tag{73a}$$

And

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \tag{73b}$$

Thus it is characterized by three parameters: the two self-inductances L_1 and L_2, and the mutual inductance M. The total energy delivered to the transformer from external sources is

$$\begin{aligned} E(t) &= \int_{-\infty}^{t} [v_1(x)i_1(x) + v_2(x)i_2(x)]dx \\ &= \int_0^{i_1} L_1 i_1' di_1' + \int_0^{i_1,i_2} M d(i_1' i_2') + \int_0^{i_2} L_2 i_2' di_2' \\ &= \frac{1}{2}(L_1 i_1^2 + 2M i_1 i_2 + L_2 i_2^2) \geq 0 \end{aligned} \tag{74}$$

It is easy to show that the last line will be non-negative if

$$\frac{M^2}{L_1 L_2} = k^2 \leq 1 \tag{75}$$

Since physical considerations require the transformer to be passive, this condition must apply. The quantity ***k*** is called the *coefficient of coupling*. Its maximum value is unity for a closely-coupled transformer.

A transformer for which the coupling coefficient takes on its maximum value $k = 1$ is called a *perfect*, or *perfectly coupled*, transformer. A perfect transformer is not the same thing as an ideal transformer. To find the difference, turn to the transformer equations (73) and insert the perfect-transformer condition $M = \sqrt{L_1 L_2}$; then take the ratio v_1/v_2. The result will be

$$\frac{v_1}{v_2} = \frac{L_1 \dfrac{di_1}{dt} + \sqrt{L_1 L_2}\,\dfrac{di_2}{dt}}{\sqrt{L_1 L_2}\,\dfrac{di_1}{dt} + L_2 \dfrac{di_2}{dt}} = \sqrt{L_1/L_2}. \tag{76}$$

This expression is identical with $v_1 = nv_2$ for the ideal transformer† if

$$n = \sqrt{L_1/L_2}. \tag{77}$$

Next let us consider the current ratio. Since (73) involve the derivatives of the currents, it will be necessary to integrate. The result of inserting the perfect-transformer condition $M = \sqrt{L_1 L_2}$ and the value $n = \sqrt{L_1/L_2}$, and integrating (73a) from 0 to t will yield, after rearranging,

$$i_1(t) = -\frac{1}{n}\,i_2(t) + \left\{\frac{1}{L_1}\int_0^t v_1(x)\,dx + \left[i_1(0) + \frac{1}{n}\,i_2(0)\right]\right\}. \tag{78}$$

This is to be compared with $i_1 = -i_2/n$ for the ideal transformer. The form of the expression in brackets suggests the v-i equation for an inductor. The diagram shown in Fig. 8 satisfies both (78) and (76). It shows how a perfect transformer is related to an ideal transformer. If, in a perfect transformer, L_1 and L_2 are permitted to approach infinity, but in such a way that their ratio remains constant, the result will be an ideal transformer.

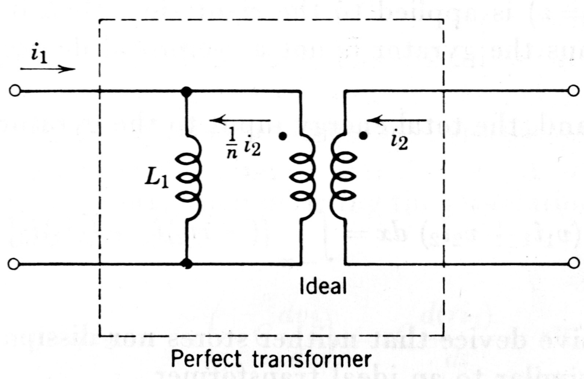


Fig. 8. Relationship between a perfect and an ideal transformer.

Losssless, memoried element.

(D) **The Gyrator**:

Definitions:

- **Port**: Two terminals, both input leads always carrying the same current.
- **Gyrator**: A two port network requiring active components for realization.

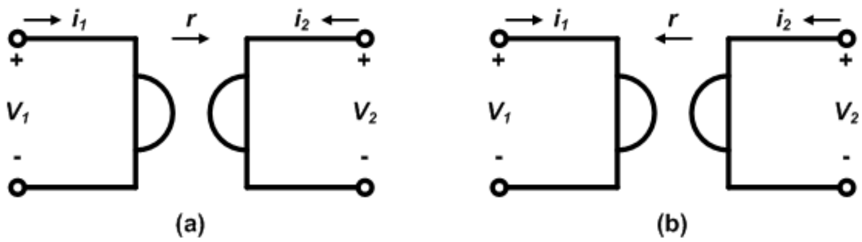


Fig. 9 A gyrator

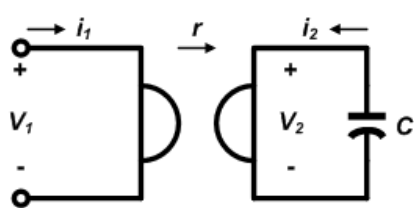
Often used to transform (convert) impedance into a different kind. Generally,

$$Z_{in} = \frac{r^2}{Z_{load}}, \ in \quad s-domain$$

For Fig. 9(a) $\begin{aligned} V_1 &= -ri_2 \\ V_2 &= ri_1 \end{aligned}$ or $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$ (79a)

For Fig. 9(b) $\begin{aligned} V_1 &= ri_2 \\ V_2 &= -ri_1 \end{aligned}$ or $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$ (79b)

$$E(t) = \int_{-\infty}^{t} (v_1 i_1 + v_2 i_2) dx = \int_{-\infty}^{t} [(-ri_2)i_1 + (ri_1)i_2] dx = 0 \quad (80)$$



Lossless, but

Nonreciprocal

Fig. 11 Gyrator terminated in a capacitor C

$i_2 = -C\frac{dv_2}{dt}$. Therefore, upon inserting the v-i relations associated with the gyrator, we observe that

$$v_1 = -ri_2 = -r\left(-C\frac{dv_2}{dt}\right) = rC\frac{d(ri_1)}{dt} = r^2C\frac{di_1}{dt} = L\frac{di_1}{dt} \quad (82)$$

(The first one is more practical, using transconductors)

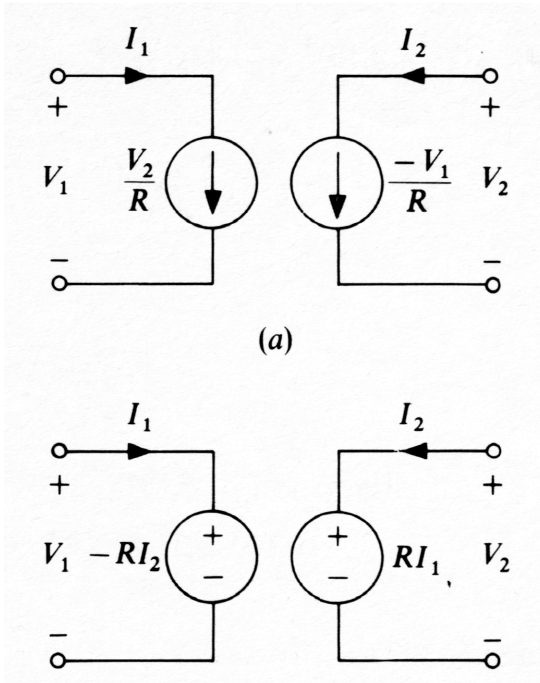


Figure 7-18 Ideal gyrator circuit

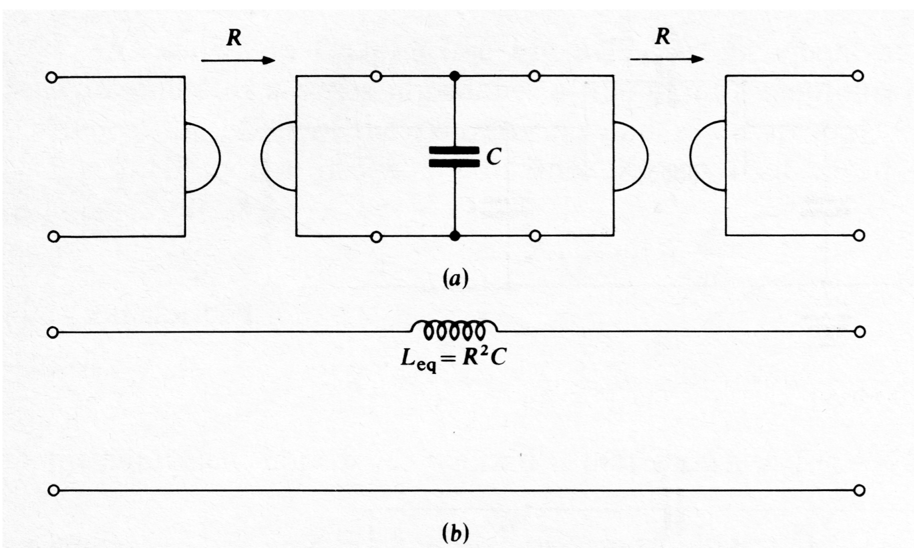


Figure 7-24 Floating-inductor simulation using gyrator

The Riordan circuit using two op-amps:

Riordan GIC/GII: general impedance converter or inverter

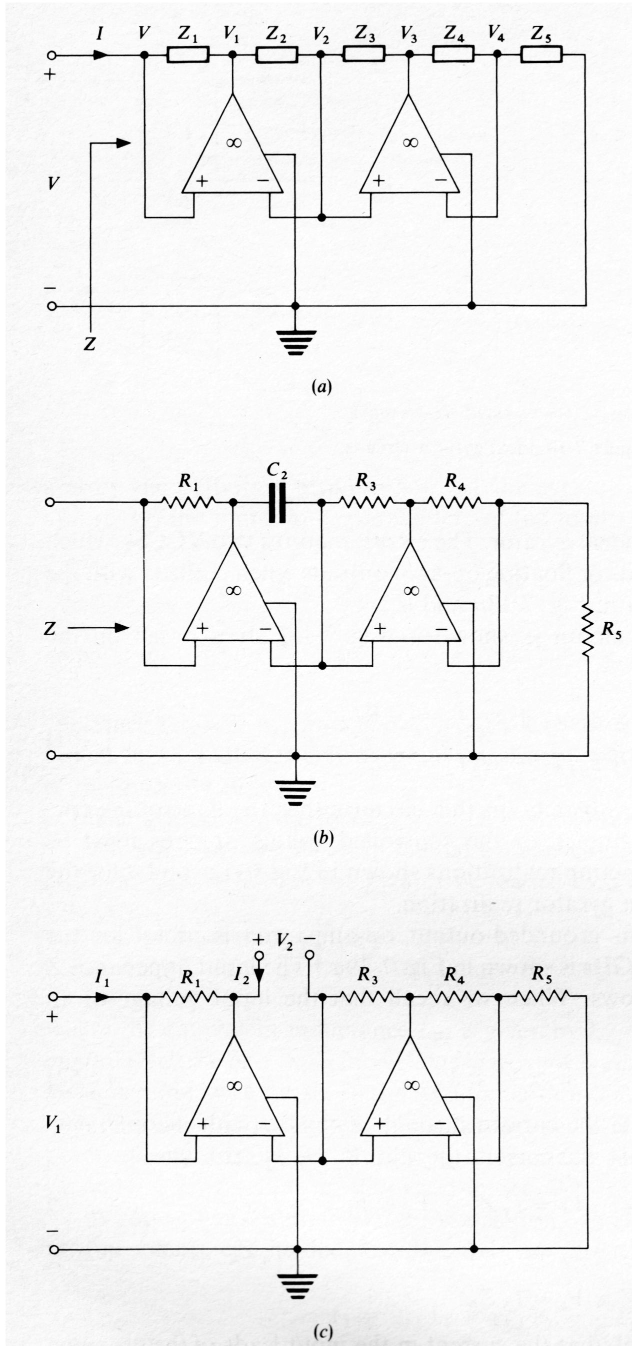


Figure 7-19 The Riordan circuit: (a) basic circuit;

(b) used as an inductor; (c) used as a gyrator

A circuit which uses two grounded-output op-amps and is useful for the realization of either GICs or GIIs is shown in Fig. 7-19*a*.† The input impedance Z can easily be found, as follows. When we recall that the input voltage of an op-amp is very nearly zero,

$$V \approx V_2 \approx V_4 \tag{7-62}$$

is obtained. Also, if we denote the current through Z_1 by I_1 (with the reference direction pointing left to right), the current through Z_2 by I_2, etc., clearly

$$\begin{aligned} I_1 &\approx I & V - V_1 &= I_1 Z_1 \approx V_2 - V_1 = -I_2 Z_2 \\ I_3 &\approx I_2 & V_2 - V_3 &= I_3 Z_3 \approx V_4 - V_3 = -I_4 Z_4 \\ I_5 &\approx I_4 & V &\approx V_4 = I_5 Z_5 \end{aligned} \tag{7-63}$$

Here we assumed, as usual, that the current in the input leads of the op-amps is zero.

Working backward in (7-63) leads to

$$V \approx I_5 Z_5 \approx I_4 Z_5 \approx -I_3 \frac{Z_3}{Z_4} Z_5 \approx -I_2 \frac{Z_3}{Z_4} Z_5 \approx I_1 \frac{Z_1}{Z_2}\frac{Z_3}{Z_4} Z_5 \approx I \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} \tag{7-64}$$

Hence

$$Z = \frac{V}{I} \approx \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} \tag{7-65}$$

If Z_5 is regarded as a load impedance, the circuit behaves like a GIC; (7-46) takes the form

$$Z(s) = f(s) Z_5(s) \qquad f(s) \equiv \frac{Z_1(s) Z_3(s)}{Z_2(s) Z_4(s)} \tag{7-66}$$

If, for example, $Z_1 = R_1$, $Z_2 = 1/sC_2$, $Z_3 = R_3$, $Z_4 = R_4$, and $Z_5 = R_5$ (Fig. 7-19*b*), then $f(s) = R_1 R_3/[(1/sC_2)R_4]$ and

$$Z = \frac{R_1 R_3}{(1/sC_2)R_4} R_5 = s \frac{R_1 C_2 R_3 R_5}{R_4} \tag{7-67}$$

Hence, the input impedance is that of an *inductor*, with an equivalent inductance value $L_{eq} = R_1 C_2 R_3 R_5 / R_4$.

As (7-67) suggests, and as can be directly verified from (7-65), the two-port formed by regarding the terminals of Z_2 as an output port is a *gyrator* if all other impedances are purely resistive (Fig. 7-19*c*). More generally, if the terminals of Z_5 (or Z_1 or Z_3) constitute the output port, the circuit of Fig. 7-19*a* is a GIC; if the terminals of Z_2 (or Z_4) form the output port, the resulting two-port is a GII.

Assume now that we choose Z_2 and Z_4 as capacitive and Z_1, Z_3, and Z_5 as resistive impedances. Then (7-65) gives, for $s = j\omega$,

$$Z(j\omega) = \frac{R_1 R_3 R_5}{(1/j\omega C_2)(1/j\omega C_4)} = -\omega^2 R_1 C_2 R_3 C_4 R_5 \tag{7-68}$$

We note that $Z(j\omega)$ is pure real, negative, and a function of ω. Such an impedance† is called a *frequency-dependent negative resistance* (FDNR). A slightly different form of FDNR can be obtained, e.g., by choosing Z_1 and Z_3 as capacitors and Z_2, Z_4, and Z_5 as resistors. Then

$$Z(j\omega) = -\frac{R_5}{C_1 R_2 C_3 R_4} \frac{1}{\omega^2} \tag{7-69}$$

As we shall see later, FDNRs are very useful for the design of active filters.

Graph Theory, Topological Analysis

- Topological Analysis: General, systematic, suited for CAD.
- Graph: Nodes and directed branches, describes the topology of the circuit, ref. direction.
- Tree: Connected subgraph containing all nodes but no loops.
- Branches in tree: twigs.
- Branches not in tree: links
- Links: Cotree

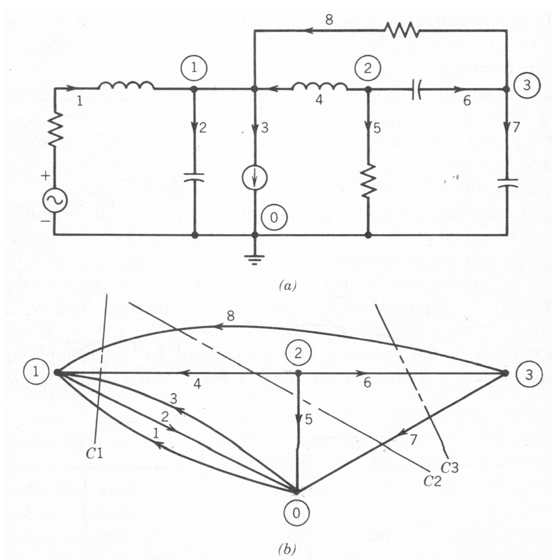


Fig. 2.2 (a) Linear circuit (b) corresponding linear directed graph

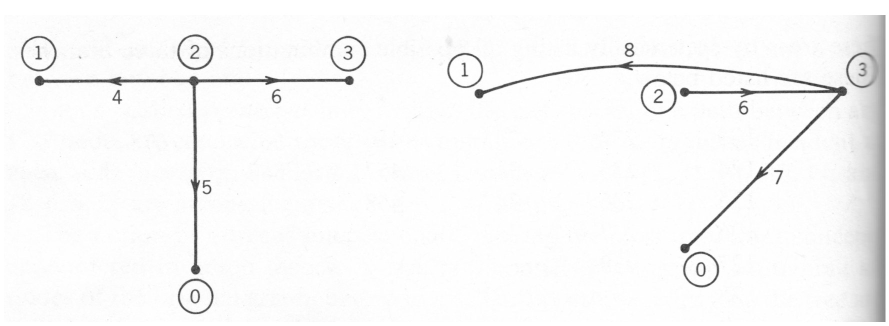


Fig. 2.3 Two of the 32 trees in the graph of Fig. 2.2b

<u>Incidence Matrix A</u>: Describes connectivity between nodes and branches.

Rule:

$$a_{ij} = \begin{cases} +1, & if \quad branch \quad j \quad is \quad directed \quad away \quad from \quad node \quad i \\ -1, & if \quad branch \quad j \quad is \quad directed \quad toward \quad node \quad i \\ 0, & if \quad branch \quad j \quad is \quad not \quad incident \quad with \quad node \quad i \end{cases}$$

As an example, the node-to-branch incidence matrix for the graph of Fig. 2.2b is

$$A_A = \begin{array}{c} \begin{matrix} (1) & (2) & (3) & (4) & (5) & (6) & (7) & (8) \end{matrix} \\ \begin{bmatrix} -1 & 1 & 1 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 \\ 1 & -1 & -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \begin{matrix} (1) \\ (2) \\ (3) \\ (0) \end{matrix} \end{array}$$

Augmented incidence matrix: contains reference node (0).

<u>Row: Nodes; Column Branches</u>

One row may be omitted, since sum of entries in each column is zero. (Reference node omitted.)

Resulting matrix: $\underline{A}$. # of non reference nodes N ≤ # of branches B → rank of $\underline{A}$ ≤ N.

Partitioned incidence matrix: Choose a tree, put its twigs in the first N columns of $\underline{A}$. Then

$$\underline{A} = [\underline{A}_t \mid \underline{A}_c] \quad \text{tree} \mid \text{cotree}$$

It can be shown that det{A_t} = ±1; and that det{$\underline{AA}_t$} = # of trees.

This proves that rank $\underline{A}$ = N! Largest singular submatrix N x N.

$\underline{Mx} = \underline{y}$, $\quad \underline{x} = \underline{M}^{-1}\underline{y}$

EXAMPLE 2. In the graph of Fig. 2.2*b*, select the tree defined by branches {2, 6, 8}. Then, using (2.3), A is written

$$A = \begin{array}{c} \begin{matrix} (2) & (6) & (8) & (1) & (3) & (4) & (5) & (7) \end{matrix} \\ \left[\begin{array}{ccc|ccccc} 1 & 0 & -1 & -1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{matrix} (1) \\ (2) \\ (3) \end{matrix} \end{array}$$

for which A_t is seen to be

$$A_t = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Graph Definitions

These trees can be found by systematically listing possible combinations of the three branches. These are listed below.

123	234	345	456	567	678
124	235	346	457	568	
125	236	347	458		
126	237	348			
127	238				
128					
134	145	156	167	178	
135	146	157	168		
136	147	158			
137	148				
138					
245	256	267	278	356	
246	257	268		357	
247	258			358	
248					
367	378	467	478	578	
368			468		

Each entry in the list must now be scrutinized to see if it contains all nodes and no loops.

TABLE 2.1
Trees and Cotrees for Graph of Fig. 2.2*b*

Trees		Cotrees	
345	246	12578	13578
347	246	12568	13568
348	248	12567	13567
456	256	12378	13478
457	257	12368	13468
458	258	12367	13467
568	267	12347	13458
678	268	12345	13457
146	356	23578	12478
147	357	23568	12468
148	358	23567	12467
156	367	23478	12458
157	368	23468	12457
158	467	23467	12358
167	478	23458	12356
168	578	23457	12346

Branch-to-Node Voltage Transformation: (KVL)

Branch Voltage: $V^t = [v_1 v_2 .. v_b]$

Node Voltage: $E^t = [e_1 e_2 .. e_n]$

By KVL, if branch k goes from node I to node j, so $a_{ik} = 1$ and $a_{jk} = -1$, then

$$V_k = e_i - e_j = a_{jk} e_i + a_{jk} e_j = [k^{th} \quad column \quad of \quad A]^t \cdot \underline{E} = a_{ik} e_i + \ldots + a_{Nk} e_N$$

In general, $\underline{V} = \underline{A^t E}$.

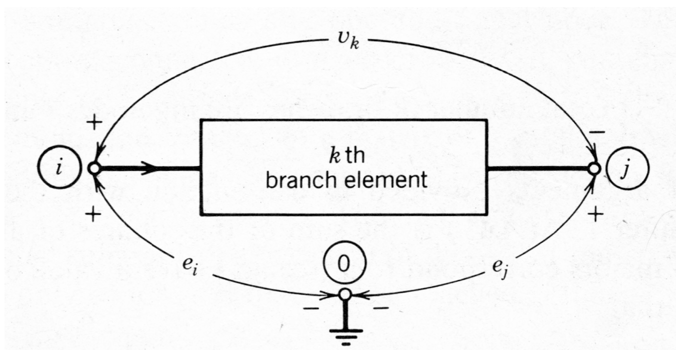


Fig. 2.4 Schematic for definition of branch and node voltages

Branch voltages expressed in terms of node voltages → there are fewer!
Purpose: formulate smallest set of linear equations before solving them.

The KCL in Topological Formulation

The KCL says that the sum of currents leaving any node is zero. Since $a_{ij} = 1(-1)$ means branch j leaves (enters) node i, the KCL for node i means

$$\sum_j a_{ij} \cdot i_j = 0 \text{ or } [i^{th} \text{ row of } \underline{A}]\underline{I} = 0,\ i = 1, \ldots, N. \text{ Hence, } \underline{AI} = 0.$$

Choose a tree, and partition $\underline{A}$ and $\underline{I}$ so that $\underline{A} = \{\underline{A}_t \mid \underline{A}_c\}$ and $\underline{I}^t = \{\underline{I}_t \mid \underline{I}_c\}$. Then $\underline{A}_t \underline{I}_t + \underline{A}_c \underline{I}_c = 0$ and $\underline{I}_t = -(\underline{A}_t)^{-1} \underline{A}_c \underline{I}_c$. This gives the twig currents from $\underline{A}$ and the link current. Note that $\underline{A}_t$ cannot be singular.

Example:

Example 4. With reference to Example 2,

$$A_t = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

whence

$$A_t^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Then,

$$A_t^{-1}A_c = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

With

$$I_t^T = \begin{bmatrix} i_2 & i_6 & i_8 \end{bmatrix}$$

$$I_c^T = \begin{bmatrix} i_1 & i_3 & i_4 & i_5 & i_7 \end{bmatrix}$$

(2.16) produces

$$i_2 = i_1 - i_3 - i_5 - i_7$$

$$i_6 = -i_4 - i_5$$

$$i_8 = -i_4 - i_5 - i_7$$

Twig currents can be found from link current. Fewer twigs than links.

These equations corroborate completely the branch current relationships exemplified in the circuit of Fig. 2.2a. (p.12)

Generalized Branch Relations

General branch for lumped linear network contains a (single) element b_k which may be an R, L, C, and dependent sources, as well as a voltage and a current source which may include the representation of initial energy stored in b_k:

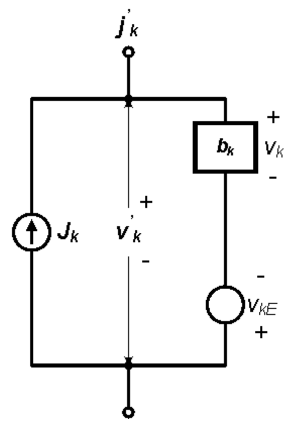


Fig. 3.1 Generalized schematic representation of *k* th branch in linear circuit

Since $i_k^{'} = i_k - J_k$ and $v_k^{'} = v_k - V_{kE}$, for the branch vectors $I^{'} = I - J$ and $V^{'} = V = V_E$ hold.

Nodal Analysis

Combining the branch relatons with the KVL ($\underline{V}^{'} = \underline{A}^t\text{E}$) and KCL ($\underline{AI}^{'} = \underline{0}$) the matrix relations

$$\text{(1)} \qquad \underline{V} = \underline{V_E} + \underline{A}^t\underline{E}$$
$$\text{(2)} \qquad \underline{AI} = \underline{AJ}$$

Result 2 equations, 3 unknown vectors: $\underline{V}$, $\underline{I}$, $\underline{E}$.

Let the V-I relations of the b_k elements be described by the matrix relation $\underline{I} = \underline{YV}$, where the diagonal element y_{ii} of $\underline{Y}$ represents the internal admittance of b_i in branch $\underline{I}$, and the off-diagonal one $y_{kl} = \frac{i_k}{v_l}$ represents a dependent I source in the branch k controlled by branch V_2. Combining (1), (2) and $\underline{I} = \underline{YV}$, and eliminating $\underline{V}$ and $\underline{I}$, in the Laplace domain, the nodal equations $\underline{Y}_N(s)\underline{E}(s) = \underline{J}_N(s)$ result, where $\underline{Y}_N(s) = \underline{A}\underline{Y}(s)\underline{A}^t$ is the N x N <u>nodal admittance matrix</u>, and $\underline{J}_N(s) = \underline{A}[\underline{J}(s) - \underline{Y}(s)V_E(s)]$ the equivalent <u>nodal current excitation vector</u>. (Due to independent sources J_k and V_{ke}.)

Node analysis parameters

Branch element voltage, currents	$v_k,\ i_k \rightarrow \underline{V},\ \underline{I}$
Branch voltages, currents	$v_k',\ i_k' \rightarrow \underline{V}',\ \underline{I}'$
Source voltages, currents	$V_{KE},\ J_K \rightarrow \underline{V_E},\ \underline{J}$
Branch admittances, branch admittance matrix	$y_{ij} \rightarrow \underline{Y}$
Nodal admittances, nodal admittance matrix	$y_{ijN} \rightarrow \underline{Y_N}$
Nodal current excitations, n. c. e. vector	$J_{iN} \rightarrow \underline{J_N}$
Node voltages	$e_i \rightarrow \underline{E}$

Example:

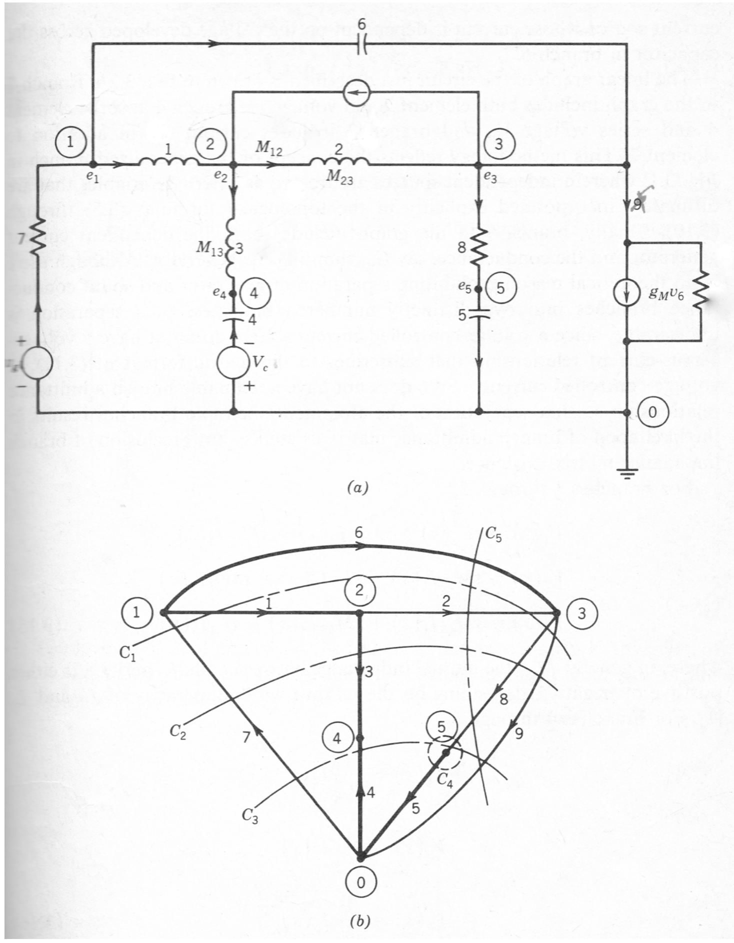


Fig. 3.2 (a) Circuit used to exemplify $\overline{Z}$ and $\overline{Y}$ matrices (b) Graph of circuit

EXAMPLE 2. For the graph of Fig. 3.2*b*,

$$A = \begin{array}{c} \\ \end{array}\overset{\begin{matrix}(1) & (2) & (3) & (4) & (5) & (6) & (7) & (8) & (9)\end{matrix}}{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix}} \begin{matrix}(1)\\(2)\\(3)\\(4)\\(5)\end{matrix}$$

In the interest of mathematical simplicity, let all $M_{ij} = 0$. Then from (3.29), (3.34), (3.35), and (3.26),

$$Y(s) = \begin{matrix}(1)\\(2)\\(3)\\(4)\\(5)\\(6)\\(7)\\(8)\\(9)\end{matrix} \overset{\begin{matrix}(1) & (2) & (3) & (4) & (5) & (6) & (7) & (8) & (9)\end{matrix}}{\begin{bmatrix} \Gamma_1/s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Gamma_2/s & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Gamma_3/s & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & sC_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & sC_5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & sC_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & G_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & G_8 & 0 \\ 0 & 0 & 0 & 0 & 0 & g_M & 0 & 0 & G_9 \end{bmatrix}}$$

It follows that

$$AY(s) = \begin{bmatrix} \Gamma_1/s & 0 & 0 & 0 & 0 & sC_6 & -G_7 & 0 & 0 \\ -\Gamma_1/s & \Gamma_2/s & \Gamma_3/s & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\Gamma_2/s & 0 & 0 & 0 & (g_M - sC_6) & 0 & G_8 & G_9 \\ 0 & 0 & -\Gamma_3/s & -sC_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & sC_5 & 0 & 0 & -G_8 & 0 \end{bmatrix}$$

and since

$$A^T = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$Y_N(s) = AY(s)A^T$ is as submitted below:

$$Y_N(s) = \begin{bmatrix} (G_7 + sC_6 + \Gamma_1/s) & -\Gamma_1/s & -sC_6 & 0 & 0 \\ -\Gamma_1/s & (\Gamma_1 + \Gamma_2 + \Gamma_3)/s & -\Gamma_2/s & -\Gamma_3/s & 0 \\ (g_M - sC_6) & -\Gamma_2/s & (G_8 + G_9 - g_M + sC_6 + \Gamma_2/s) & 0 & -G_8 \\ 0 & -\Gamma_3/s & 0 & (sC_4 + \Gamma_3/s) & 0 \\ 0 & 0 & -G_8 & 0 & (G_8 + sC_5) \end{bmatrix}$$

The nodal current vector is obtained through use of (3.43). With

$$J^T(s) = \left[0 \; I_g(s) \; 0\,0\,0\,0\,0\,0\,0\right]$$

and

$$V_E^T(s) = \left[0\,0\,0 \; -V_c(s) \; 0\,0 \; V_g(s) \; 0\,0\right]$$

$$Y(s)V_E(s) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -sC_4V_c(s) \\ 0 \\ 0 \\ G_7V_g(s) \\ 0 \\ 0 \end{bmatrix}$$

and, thus,

$$J(s) - Y(s)V_E(s) = \begin{bmatrix} 0 \\ I_g(s) \\ 0 \\ sC_4V_c(s) \\ 0 \\ 0 \\ -G_7V_g(s) \\ 0 \\ 0 \end{bmatrix}$$

Finally,

$$J_N(s) = \begin{bmatrix} G_7V_g(s) \\ I_g(s) \\ -I_g(s) \\ -sC_4V_c(s) \\ 0 \end{bmatrix}$$

It is understood that

$$J_N(s) = Y_N(s)E(s)$$